1.2 Inverses Notes

Inverse Relations and Inverse Functions –

The ordered pair (*a*, *b*) is in a relation if and only if the ordered pair (*b*, *a*) is in the inverse relation.

Horizontal Line test – The inverse of a relation is a function if and only if each horizontal line intersects the graph of the original relation in at most one point.

See examples on pg. 131

A function whose inverse is a function has a graph that passes both the horizontal and vertical line tests. Such a function is **one-to-one**, since every *x* is paired with a unique *y* and every *y* is paired with a unique *x*.

Inverse function: If *f* is a one-to-one function with domain *D* and range *R*, then the **inverse function of f denoted f^{-1}** is the function with domain *R* and range *D* defined by

$f^{-1}(b) = a$ if and only if $f(a) = b$.

Finding the inverse function – switch *x* and *y* then solve for *y*.

Ex. Find an equation for $f^{-1}(x)$ if $f(x) = 2x + 5$. Then find the domain of f^{-1}, including any restrictions "inherited" from *f.*

Note: Many functions are not one-to-one, therefore they will not have an inverse.

The Inverse Reflection Principle: The points (*a,b*) and (*b,a*) in the coordinate plane are symmetric with respect to the line *y* = *x*. The points (*a,b*) and (*b,a*) are **reflections** of each other in the line *y* = *x* .

Finding an inverse function graphically: See example 5 pg. 132.

Inverse composition Rule: A function *f* is one-to-one with inverse function g if and only if

f(g(x)) = x for every x in the domain of g,

and g(f(x)) = x for every x in the domain of f.

Verifying Inverse Functions

Show algebraically that $f(x)=x^3+1$ and $g(x)=\sqrt[3]{x-1}$.

(Remember, if $f(g(x))=x$ and if $g(f(x))=x$, then they are inverses.)

Another example of finding an inverse function:

Show that $f(x)=\sqrt{x+3}$ has an inverse function and find a rule for $f^{-1}(x)$. State any restrictions on the domains of f and f^{-1}.